Black, E. E. A test to discover where pupils make mistakes in word problems in algebra. 1947

A TEST TO DISCOVER WHERE PUPILS MAKE MISTAKES IN WORD PROBLEMS IN ALGEBRA

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A TEST TO DISCOVER WHERE PUPILS MAKE MISTAKES IN WORD PROBLEMS IN ALGEBRA

THESIS

Submitted by

Edwin Ernest Black

(B.S. in Ed., Gorham State Teachers College, 1946)

In partial fulfillment of requirements
for the degree of Master of Education

First Reader: Henry W. Syer, Assistant Professor of Education

Second Reader: William C. Kvaraceus, Assistant Professor of Education

Third Reader: Roy O. Billett, Professor of Education

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FIGURE 1 - FREQUENCY OF SCORES 39

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FIGURE I - FREQUENCY OF SCORES 35

INTRODUCTION

It has been the experience of the writer that students in algebra have difficulty in solving verbal problems. Since the author's teaching experience has been limited to first-year algebra, he cannot say that this is true for more advanced students. However, from conversations with other teachers, he has been led to believe that this difficulty is also found in the upper classes of high school mathematics.

Although material pertinent to the study of difficulties in solving word problems is available, the author believes that treatment on the subject as a whole has been neglected. Most of the previous work done on this phase of education has been concerned with the various steps which go to make up the solution of verbal problems. For example, Daily[1/] has a very good treatment on selection of essential data in word problems. More attention will be paid to these reports in the first chapter of this thesis.

It is the primary purpose of the author to report in this thesis on the results of a test constructed to discover where the pupils are making mistakes in their attempts to solve verbal problems. A secondary purpose is to briefly consider the reasons for the use of word problems in elementary algebra. This latter aim will be accomplished by reviewing one of the reports available.

1/ Daily, Benjamin W., The Ability of High School Pupils to Select Essential Data in Solving Problems, Teachers College, Columbia University, New York City, 1925.

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It has been the experience of the writer that students in algebra have difficulty in solving verbal problems. Since the author's teaching experience has been limited to first-year algebra, he cannot say that this is true for more advanced students. However, from conversations with other teachers, he has been led to believe that this difficulty is also found in the upper classes of high school mathematics.

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1/ Duncan, Benjamin W., The Ability of High School Pupils to Select Essential Data in Solving Problems, Teachers College, Columbia University, New York City, 1933.

The material upon which this thesis is based was obtained by giving a newly constructed and original algebra test to 462 second-year algebra students of Massachusetts and Vermont. A copy of this test is included in the appendix.

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CHAPTER I

REVIEW OF RELATED LITERATURE

From the literature which is pertinent to the subject, the writer has chosen several representative works which will be briefly summarized.

There was no report discovered which treats the topic of problem solving in algebra in its entirety. However, several works were chosen to represent various sub-topics of the subject being considered in this thesis.

Section A of this chapter is given over to the topic of why problems are taught. The section is mainly a condensation of Chapter I of Sanford's[1/] "History and Significance of Problems in Algebra".

A. WHY PROBLEMS ARE TAUGHT

I. The Three Types of Problems

Algebraic problems may be grouped into three general divisions. These are (1) genuine problems, (2) puzzles, and (3) an intermediate class.

Genuine problems are those which arise from our environment and are used to solve real situations. They are included in our textbooks in an attempt to link reality to theory. This justifies the inclusion of the subject matter on one hand, and shows how mathematics is useful in the life we

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A. WHY PROBLEMS ARE TAUGHT

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Algebraic problems may be grouped into three general divisions. These are (1) genuine problems, (2) puzzles, and (3) artificial types.

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lead on the other hand.

An example of a genuine problem is, "A dealer has 100 gallons of mixture which is 80 per cent alcohol. How many gallons of a 90 per cent mixture must be added so as to make a mixture which is 82.5 per cent alcohol?"

Puzzle problems are due entirely to man's curiosity and to the liking which he has for riddles. They exist mainly because of the appeal they have to our imagination. They are placed in textbooks in order to show some of the intrinsic values of mathematics, and to satisfy the interest man harbors for puzzles.

An example of a puzzle problem is, "A man is twice as old as his wife was when he was as old as she is now. When she is as old as he is now, the sum of their ages will be 100 years. Find their ages now."

The third type of word problem is that which might be called the pseudo-real problem. This type of problem attempts to combine the usefulness of the real problem with the fascination of the puzzle problem. The intermediate problem has more possibilities for variations than has the genuine problem. Another reason for their existance is the desire to experiment, that is to see if first one and then another of the stated quantities may be found. The last reason is the purely negative reason that no other problems have been devised to fir the situation to better advantage.

An example of a pseudo-real problem is, "James has twice as much money as Charles and four times as much as William. Together they have $1.75. How many cents has each?"

In this report, the author has attempted to deal with some of each of these types. This is done so that the student taking the test will not be working entirely on ground with which he may be familar or which may be disliked.

II. The Use of Problems in Textbooks

Various authors give more attention to certain types of problems then do others. Also with the change of time there has been a change in the problems which appear in the textbooks. Thompson[1/] and Poole[2/] analyzed nine American textbooks published from 1808 to 1929. They discovered the number of times the various types of problems appear. Thse results are given in the following paragraphs.

Thompson found twelve types of problems occurring in one unknown. In order of frequency these are: numbers, distribution, uniform motion, mixture, geometric, investment, work, age, money, cost, percentage, value, and miscellaneous.

1/ Thompson, Therm B., "A Study of the Variations in Type and Frequency of Written Problems in One Unknown; as Evidenced by an Analysis of Several Standard Algebra Textbooks", Unpublished Master's Thesis, Boston University, School of Education, 1933.

2/ Poole, Roger K., "A Study of the Variations in Type and Frequency of Written Problems in Two Unknowns; as Evidenced by an Analysis of Several Standard Algebra Textbooks", Unpublished Master's Thesis, Boston University, School of Education, 1933.

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2/ Poole, Roger K., "A Study of the Variations in Type and Frequency of Written Problems in Two Unknowns; as Evidenced by an Analysis of Several Standard Algebra Textbooks", Unpublished Master's Thesis, Boston University, School of Education, 1933.

Poole also found twelve types of problems occurring in two unknowns. In order of frequency these are: numbers, distribution, uniform motion, mixture, cost, geometric, money, age, investment, weight, capacity, and value.

III. Reasons for Teaching Word Problems

There are several reasons given for the teaching of word problems. Among these are three which the author will discuss briefly. They are (1) as an exercise in analysis, (2) they link theory with reality, and (3) interest of the pupils in word problems.

Among the reasons given for using verbal problems as an exercise in analysis is to provide practice in analyzing situations to see what mathematical ralationships are present and how these relationships may be used to answer a question stated in the problem. The other main reason is to provide for the transfer of training, in so far as the pupil sees likenesses between situations.

In order to justify the inclusion of the subject matter and to show how this theory is useful in life situations, the attempt to link reality with mathematics has been made. Genuine or real problems are the medium for this although at times the use of puzzles and pseudo-real problems are used to convey this information.

The third reason is pupils' interests. Many pupils are more interested in working word problems than in doing other

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types of algebraic computation. Downey[1] discovered that problems ranked fourth in the preference of high school students. The three most popular interests were (1) equations, (2) the four fundamental operations, and (3) factoring. Thus, it seems that pupils are interested in word problems and this is a good reason for their inclusion in mathematics courses.

B. REVIEW OF OTHER LITERATURE

In this section the writer will briefly summarize the findings of several students on various phases of problem solving. One of these reports does not actually deal with problem solving, but is included since it is connected with the subject of this thesis.

Powell[2] discovered that teachers overestimate their ability to create interest in problem-solving. More problems were rated higher in interest by the teacher than by the pupils. "The pupil is most interested in problems that express activity; that describe situations in which he may participate".[3] The pupil likes a problem better when it is stated so as to give a definite challenge.

Drake[4] and cooperating teachers, gave vocabulary tests covering algebraic words and expressions to students at the

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1/ Downey, Walter F., "Mathematical Interests of High School Pupils", Report of the National Committee On Mathematical Requirements, 1923.

2/ Powell, Jesse J., A Study of Problem Material in High School Algebra, Teachers College, Columbia University, New York City, 1929.

3/ Powell, Jesse J., Ibid, pp 33.

4/ Drake, Richard M., "The Effect of Teaching the Vocabulary of Algebra", Journal of Educational Research, April 1940, pp 601-610.

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Drake[4] and cooperators tested a five vocabulary tests covering algebraic words and expressions to students at the

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1/ Downey, Walter S., "Attempts to find interests of high school pupils", Report of the National Council Teachers of Mathematics Department, 1929.

2/ Powell, Jesse A., A Study of Problem Material in High School Algebra, Columbia Teachers College, Columbia University, New York City, 1930.

3/ Powell, Jesse A., Ibid. pp 23.

4/ Drake, Richard M., "The Effect of Teaching the Vocabulary of Algebra", Journal of Educational Research, April 1940, pp 601-610.

beginning and end of each of six units of work. Seven schools in and around Minneapolis, Minn. cooperated in the study. Each school provided two groups, a vocabulary group and a control group. The vocabulary group put special emphasis on the mastery of frequently missed words as discovered by the pre-test. The other group received such instruction incidently. At the end of the study, the vocabulary group showed the most favorable effect upon algebraic achievement. No effort was made to see if this gain in vocabulary understanding was lasting or temporary.

McKim's[1/] study was undertaken to discover the relationships between the reading of algebraic and non-algebraic materials, and between these measures and achievement in algebra. Two tests were built, one on reading problems in algebra, and one on reading passages similar to those met in algebra textbooks. Three other tests were also given--Terman Group Test of Mental Ability-Form A, New Stanford Reading Test-Form V, and the Orleans Algebra Prognosis Test. Correlations were obtained between all these tests and the results on the mid-term and final examinations given in the school. All the correlations were smaller for the final test then for the mid-term test. The correlations obtained are given in the accompanying table. It is significant to note that they are all fairly high and show that algebraic achieve-

1/ McKim, Margaret G., The Reading of Verbal Material in Ninth Grade Algebra, Teachers College, Columbia University, New York City, 1941.

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1/ Mellin, Margaret C., The Reading of Verbal Material in Minne Units Algebra. Teachers College, Columbia University, New York City, 1931.

ment is partially dependent upon reading ability.

Correlations obtained by McKim[1/]

Test	Mid-Term	Final
Terman Group Test of Mental Ability-Form A	.52	.46
New Stanford Reading Test-Form V		
Paragraph Reading	.47	.40
Word Meaning	.37	.36
Orleans Algebra Prognosis Test	.56	.47
Test of Reading Problems*		
Parts A and B	.50	.49
Part C	.38	.32
Test of Reading Passages*		
Part A	.56	.52
Part B	.62	.61

*Tests built by McKim for the study

The purpose of Clark's[2/] report was to determine (1) the improvement made by ninth-grade algebra pupils in their general reading ability, due to intensive remedial instruction; and (2) the improvement which this may effect in the algebraic achievement of these same pupils. Two groups were given the remedial instruction, one group did not receive any remedial instruction.

There was a mean gain of 2.7 in grade level for the remedial group, and 1.8 for the non-remedial group. The correlation between reading comprehension and algebraic achievement was .70 for the remedial group and .45 for the non-remedial group. This shows that a higher coefficient of correlation between reading comprehension and algebraic

1/ McKim, Margaret G., Ibid, pp 71.
2/ Clark, Mildred I., "The Effect of Remedial Work in Reading Upon Algebraic Achievement", The Mathematics Teacher, February 1939, pp 65-67.

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Correlations obtained by Zeller[1]

Test	Mid-Term	Final
Iowa Group Test of General Ability-Form A	.55	.46
New Standard Reading Test-Form T		
Paragraph Meaning	.47	.40
Word Meaning	.37	.36
Yoakum Algebra Progressive Test	.55	.47
Test of Reading Problems*		
Parts A and B	.53	.45
Part C	.38	.35
Test of Reading Passages*		
Part A	.55	.32
Part B	.35	.21

*Data taken by Malin for the study

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1/ Zeller, Margaret S., Ibid., pp 71.

2/ Stark, Albert L., "The Effect of Remedial Work in Reading Upon Algebraic Achievement", The Math-ematics Teacher, February 1957, pp 85-87.

achievement may be obtained by remedial instruction in the former.

Daily[1] found that pupils doing regular problems obtained higher scores than if doing problems containing superflurous or insufficienct data. He gave 6 regular and 6 special problems to 206 students. The scoring was done on a 2, 1, 0 basis, the 1 being for partial success. The mean score for the regular problems was 9.84 and for the special problems 3.49. From these results it is apparent that pupils are not trained in the selection of essential data.

Daily also discovered "(1) that it is not a common practice to require first-year high school pupils to exercise discrimination in the choice of data for the solution of algebra problems, (2) that it is possible, by means of specific instruction, so to teach pupils that they consciously exercise this desirable function, (3) that when pupils have been taught to select data necessary for solution of mathematical problems, they improve in ability to select data for problems in other fields of learning".[2]

1/ Daily, Benjamin W., Op. Cit.
2/ Daily, Benjamin W., Ibid, pp 81.

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1/ Daily, Benjamin H., op. cit.
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CHAPTER II

BUILDING THE TEST

Once the problem was determined, the testing instrument remained to be chosen. Since there was no satisfactory commercial test available to fill the needs of the problem it became necessary to construct a test that would fulfill the specifications set up.

A. DISCRIPTION OF THE TEST

The present test was decided upon after carefully investigating other possibilities and other test items. Mr. Syer's class in "The Teaching of Secondary School Mathematics" took the tentative test and made many valuable suggestions which helped a great deal in revising the test. A large number in this class were mathematics teachers or had had experience in the teaching of mathematics.

The test consists of seven sections. These sections or topics were chosen since the author believes they have the most direct bearing upon the subject under consideration. Other topics were considered but were omitted as not being as pertinent to the study as those retained.

One reason for not including more topics and more items in this test is the time element. A short period of time does not allow one to test this subject with the thoroughness it deserves. By including these seven topics in one test of

CHAPTER IV

SOLVING THE TEST

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54 items it was easier to obtain the cooperation of teachers to administer the test to their classes. A period of 45 minutes is the ideal length of time to allow students to work on the test.

However, since different school systems have periods of unequal lengths, the author has set no definite time limits on the test or on its individual sections, preferring to allow the pupils to proceed according to their individual speeds and abilities. Thus the students in classes with short class periods can go from one section to another and better results should be obtained. This procedure should also give a better indication as to where difficulty lies in the solving of word problems in algebra.

By allowing the students to do those problems which they can do most easily and quickly, they will leave the more difficult ones unanswered or will return to do them after completing the others. The time that they save in this way allows them to try more of the items then they might try otherwise. This also shows where some of the difficulties of problem solving lie.

The seven sections of the present test are: vocabulary, arithmetic, algebra fundamentals, reading, setting up algebraic expressions, setting up algebraic equations, and solving algebraic equations.

The processes in the sections range from elementary addition and subtraction through quadratic equations. Thus

At times it was easier to obtain the cooperation of teachers to administer the test to their classes. A period of 45 minutes is the ideal length of time to allow students to work on the test.

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The processes in the sections range from elementary addition and subtraction through quadratic equations. Thus

the author feels that practically all pertinent facts relative to solving word problems have been included.

There are other topics which one could include in a test on word problems, but because of a limited time in which to give the test they were not included. For example, one of these topics is checking. Once the pupil has gone through the steps of problem solving and obtained an answer he should return to the problem and find out if his obtained answer satisfies the conditions stated in the problem. If his answer does satisfy these conditions he has done his work correctly, if it does not he must go over his work to discover his mistake.

B. DISCUSSION OF THE TEST

I. Vocabulary

The words making up the list contained in the vocabulary section of the test were decided upon as being among the key words in word problems in algebra. This was done by studying word problems in several algebra texts to discover which words were most often used as the "key words" in problems. The first list of words considered contained nearly seventy words pertinent to the subject of algebra and problem solving.

Many of these words were eliminated because they do not occur with enough frequency to be considered as important to the study. Others were excluded since it is possible for a pupil to perform the operation without knowing the definition

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Many of these words were eliminated because they do not occur with enough frequency to be considered as important to the study. Others were excluded since it is possible for a pupil to perform the operation without knowing the termin-ion

of the term. Still other words were omitted because they were not so common in algebra as in geometry or trigonometry. For example, angle or tangent are not as much algebraic as they are geometric and trigonometric.

Column B of Section I contains definitions or examples to match with the words of Column A. Since many teachers of mathematics teach slightly different definitions for the same term, examples instead of verbal definitions have been used whenever possible or practical. For example, in Column B, P is definitely an example of a formula and should be matched with number 9 of column A. If the writer were to define "formula" in words it might have led to more misunderstanding to those who would have other definitions for "formula".

Also, by using examples where possible the pupils can do the exercises with less time spent in reading definitions and will therefore be able to do the section faster. This gives the pupil more time to work on the remainder of the test; time which might be valuable if the class period is less than 45 minutes.

II. Arithmetic

Since it is quite probable that many of the pupils in our algebra classes have difficulty doing arithmetic, a section has been included on this subject. If it is true that some pupils have difficulty doing elementary arithmetic they will find themselves handicapped when it becomes necessary to use these processes in algebra.

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The reason for including a section on arithmetic is to discover how many pupils are bothered by a poor background in this fundamental mathematics. The section contains fractions and decimals as well as the four fundamental operations with which the pupil should be familiar.

By including these different processes it may be possible to discover which arithmatical processes are causing difficulty to high school algebra students.

III. Algebra Fundamentals

A section on the fundamentals of algebra is included in the test to discover if the pupils are having difficulty in using the fundamental skills they acquired in their first experiences in algebra. Those included were considered to be the most important ones used in algebraic equations arising from problem solving.

In giving the Hotz Algebra Scales, Form A, W. C. Hurd[1/] discovered that fractions gave pupils the most difficulty among the processes. In this study Hurd also found that equations and formulas also gave the pupils difficulty.

In this study we are interested in discovering if failure in doing the fundamental processes is related to the solving of word problems.

1/ Hurd, W.C., "A Study of Individual Differences and Failures in Algebra", Unpublished Master's Thesis, Boston University, School of Education, 1935.

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1\ Burd, W.C., "A Study of Individual Differences and Factors in Algebra". Unpublished Master's Thesis. Boston University. School of Education. 1936.

IV. Reading

In order to be successful in solving word problems, one needs to read and understand the problem. He must understand what he is to find, what pertinent information is given and how to use this information. It is important that the student should recognize when extra data are included in a problem and that he be able to reject that which is not needed to solve the problem.

Buckingham[1/] discovered that the relationship between silent reading ability and ability in first year algebra is low but positive. If this study showed correlations between reading ability and ability in solving word problems it is possible that a higher relationship might exist. In another study, Stright[2/] found that special instruction in general reading skills was reflected in proficiency in algebra. Spangler[3/] says, "From research and experience it seems clear that improvement in reading skills results in improvement in problem solving."

1/ Buckingham, Guy E., "The Relationship Between Silent Reading Ability and First Year Algebra Ability", The Mathematics Teacher, March 1937, pp. 130-132.

2/ Stright, Isaac L., "The Relation of Reading Comprehension and Efficient Methods of Study to Skill in Solving Algebraic Problems", The Mathematics Teacher, December 1938, pp. 368-372.

3/ Spangler, Mamie, "Why is the Ability to Recognize Relationships in Reading of Problems so Vital, and What Methods May be Used to Develop this Ability?", School Science and Mathematics, May 1946, pp. 450.

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1/ Zweiniggan, op. c., "The Relationship between Silent Reading Ability and First Year Algebra Ability", The Elementary School Journal, March 1937, pp. 19 - 140.

2/ Wright, Susan L., "The Relation of Reading Comprehension and Difficulty Reading of Text to Skill in Solving Algebraic Problems", The Mathematics Teacher, December 1938, pp. 326-372.

3/ Dangler, Mamie, "Why is the Ability to Recognize Relationships in Reading of Problems so Vital, and What Methods May be Used to Develop this Ability", School Science and Mathematics, May 1946, op. cit.

In this section we actually begin to do the steps of problem solving, whereas the first three sections of the test have a more indirect bearing on the subject. Reading is actually the first process the student attempts in solving verbal problems. If the student is unable to read and interpret the wording of the problem with accuracy, he will encounter difficulty in the other steps of his work. Also, if the pupil doesn't understand the meaning of a key word in a problem, difficulty will arise when he attempts a solution. For example, one may try to solve a problem with one unknown when there are two unknowns present.

The section on reading included in this test was designed to find out if the pupil could discover certain factual information contained in the items. These problems were chosen because they represent difficulties often appearing in algebraic exercises. Perhaps they seem more fifficult than the usual problem, but it is the writer's opinion, that problems like these are causing a great deal of confusion to algebra students.

V. Setting up Algebraic Expressions

Once the pupil has read the problem and decided what the question is that he must answer, his next step is to set up algebraic expressions to help him in solving the problem. This step is one of great importance in problem solving. If the pupil fails to give the correct values to the expressions needed, his work will be in error. Also the procedures

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following this important part will not be correct.

It is in this step that the pupil needs to realize the fact that he will need to set up as many unknowns as he found necessary by reading the problem. He must also be able to place correct values for each expression that is compared with some other. For example if a problem states that there are twice as many quarters as dimes in a certain situation, he must give the expression for quarters twice the value as that for dimes.

The two problems stated in this section were chosen because the author believes they present a difficulty to the student, and because problems like these occur quite frequently. Poole[1/] in his analysis of nine algebra textbooks discovered that motion problems containing two unknowns were third in quantity following numbers and distribution problems. Thompson[2/] analyzed the same nine books for word problems in one unknown. His results show that mixture problems are fourth in quantity following numbers, distribution, and motion.

The reason for not including age problems or coin problems is that they are more common and the pupils have become better acquainted with them. The student should be able to solve problems which state different situations. It seems to this writer that problems like those chosen present circumstances which the pupil must see and overcome if he is to solve the

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1/ Poole, R. K., Op. Cit.
2/ Thompson, T. B., Op. Cit.

problem successfully.

VI. Setting up Equations

After reading the problem to find out what the question is to be answered and the algebraic expressions have been proposed, the student's next operation is to place the information obtained into the form of an equation. This procedure transforms the verbal question into algebraic symbols, and when the equation is solved will provide the answer to the question. One can readily see that this step is an important undertaking.

If the pupil fails to include the correct expressions in an accurate arrangement in the equation, he will be unable to answer the question stated in the problem al all. In order to use the correct expressions, he must choose those values he has set up in the previous step which are pertinent to the situation given. At times he will not use all the expressions.

The two problems selected for this part of the test are similar to the problems of Section V, in that each division contains a distance and a mixture problem. Both problems have algebraic expressions which are not needed to set up the equation but should help clarify the other expressions.

To illustrate the last sentence, the author will give examples from each problem. In problem 1, X = number of miles given. This expression is not needed to solve the problem but helps clarify the expressions $\frac{X}{12}$ and $\frac{X}{6}$. In problem 2 only

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two of the expressions are needed to set up the equation. These are $(1/4)(60 + X)$ and $6 + X$. The others are included to help clarify the derivation of these expressions.

These two equations when set up provide the student with two types of problems. One contains fractions, the other does not. The main reason for not including a greater variety of problems in this section is the time element.

VII. Solving Equations

After the student has set up the equation, the next operation in the solution of word problems is to solve the equation in order to obtain the answer to the question asked in the statement.

The purpose of this section is two-fold. First, it presents the individual with a problem and its equation to solve. Secondly, it requires him to find out what the question is that he must answer.

At first glance, it might seem that this section is a repetition of Section III as both contain algebraic equations to be solved. However, the equations in this section are more commanding and require more of the fundamental processes than those of Section III. Here he is confronted by two quite difficult equations, one necessitates the use of the quadratic formula, the other involves the solving of simultaneous equations. The third equation contains a fraction and makes use of parentheses.

Although there are other types of equations, these

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Although there are other types of equations, these

represent those which are apt to be most familiar to the pupil. The time limit again is the reason for not including more problems in this division of the test.

CHAPTER III

CORRELATIONS AND VALIDITY

Whenever a person is looking at a test or reading a discussion of the test there are several questions that come to mind. It is the writer's purpose to answer some of those questions in this chapter.

Among frequently asked questions are: (1) Is the test valid? (2) Is the test reliable? (3) How does the test correlate with other tests? and (4) What was the group these correlations were based upon?

Before attempting to answer the first three questions, the author will first answer the fourth question. It is his belief that the reader should know about the group before the other material of this chapter is presented.

A. THE GROUP TAKING THE TEST

The test was given to 462 second-year algebra students of Massachusetts and Vermont. Six high schools gave the test to their pupils and this report is based upon the answers these pupils made. The schools and number of pupils is as follows:

Barre High School	Barre, Mass.	23
Belmont High School	Belmont, Mass.	107
Greenfield High School	Greenfield, Mass.	48
Oak Bluffs High School	Oak Bluffs, Mass.	4
Somerville High School	Somerville, Mass.	254
Spaulding High School	Barre, Vermont	26

These 462 students had all had one previous year's work

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A. THE GROUP MAKING THE TEST

The test was given to 632 seventh-year girls students of Massachusetts and Vermont. Six high schools gave the test to their pupils and this report is based upon the answers those pupils made. The schools and number of pupils is as follows:

Barre High School	Barre, Mass.	42
Belmont High School	Belmont, Mass.	107
Greenfield High School	Greenfield, Mass.	42
Dan Shurtle High School	Dan Shurtle, Mass.	2
Somerville High School	Somerville, Mass.	224
Spaulding High School	Barre, Vermont	20

These 632 students had all had one previous year's work

in algebra. Most of the pupils are members of the junior and senior classes. The ages of the group range from 14 to 29. The two years 16 and 17 include 369 members of this population. It is probable that veterans who have returned to school are responsible for the older group in second-year algebra classes. Table I shows the age and class range for the entire group.

TABLE I - AGE AND CLASS RANGE

	14	15	16	17	18	19	20	21	22	24	29	No Age	Totals
Soph.*				1									1
Jun.		30	209	48	1		1			1		5	295
Sen.			7	80	23	4	5	2				2	123
P.G.					2	1	3	2			2		10
N.C.	1		13	11		1	2	1	1			3	33
Totals	1	30	229	140	26	6	11	5	1	1	2	10	462

*In this table and the succeeding tables class names have been abbreviated. The classes are: Sophomore, Junior, Senior, Post-Graduate, and No Class Stated.

Of these 462 students, 299 are boys. Their ages range from 14 to 29. These boys represent members of junior and senior classes and contain the entire group of post-graduate students. Table II shows the age and class range for boys.

One hundred fifty-seven girls are represented in the results of the test. Their age range is considerably less than that of the boys, due perhaps to the presence of veterans in the group of boys. The girls ages range from 15 to 18.

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TABLE I - AGE AND CLASS RANGE

	14	15	16	17	18	19	20	21	22	23	24	Totals
Soph.*				1								1
Jun.		3	209	88	1		1		1		6	298
Sen.			7	80	34	4	5	2			3	135
F.G.					2	1	3	3		2		10
J.C.	1		13	11		1	2	1	1		3	33
Totals	1	3	220	180	36	6	11	6	1	1	12	482

*In this table and the succeeding tables class means have been abbreviated. The classes are: Sophomore, Junior, Senior, Post-graduate, and No Class Stated.

Of these 482 students, 300 are boys. Their ages range from 14 to 24. These boys represent members of junior and senior classes and contain the entire group of post-graduate students. Table II shows the age and class range for boys.

One hundred fifty-seven girls are represented in the results of the test. Their age range is considerably less than that of the boys, due perhaps to the presence of veterans in the group of boys. The girls also range from 15 to 18.

TABLE II - AGE AND CLASS RANGE FOR BOYS

	14	15	16	17	18	19	20	21	22	24	29	No Age	Totals
Jun.		15	129	36	1		1			1		4	187
Sen.			1	48	17	4	5	2				2	79
P.G.					2	1	3	2			2		10
N.C.	1		7	9		1	2	1	1			1	23
Totals	1	15	137	93	20	6	11	5	1	1	2	7	299

They represent members of the sophomore, junior, and senior classes. Table III shows the age and class range for the girls.

TABLE III - AGE AND CLASS RANGE FOR GIRLS

	15	16	17	18	No Age	Totals
Soph.			1			1
Jun.	14	78	12		1	105
Sen.		6	31	6		43
N.C.		6	2			8
Totals	14	90	46	6	1	157

Of the 462 pupils, 6 cannot be classified according to sex. There are two reasons for not including them in the previous divisions: (1) they did not write their names on the answer sheet, and (2) it was not possible to tell from the name whether the student is a boy or a girl. Table IV shows the age and class range for this group of unclassified students.

TABLE II - AGE AND CLASS RANKS FOR BOYS

	14	15	16	17	18	19	20	21	22	23	24	25	No Age	Totals
Jun.		15	129	36	1		1				1		4	187
Sen.			1	48	17	4	5	2					2	79
P.G.					2	1	3	4				3		10
N.G.		1		7	9	1	2	1		1			1	23
Totals		1	15	187	75	20	6	11	2	1	1	3	7	299

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TABLE III - AGE AND CLASS RANKS FOR GIRLS

	15	16	17	18	No Age	Totals
Soph.			1			1
Jun.	14	78	12		1	105
Sen.		6	21	6		43
N.G.		6	2			8
Totals	14	90	45	6	1	157

Of the 157 pupils, 6 cannot be classified according to age. There are two reasons for not including them in the previous divisions: (1) they did not write their names on the answer sheet, and (2) it was not possible to tell from the same whether the student is a boy or a girl. Table IV shows the age and class range for this group of unclassified students.

TABLE IV - AGE AND CLASS RANGE FOR UNCLASSIFIED

	15	16	17	No Age	Totals
Jun.	1	2			3
Sen.			1		1
N.C.				2	2
Totals	1	2	1	2	6

The third or junior year seems to be the most common time for students to take second-year algebra. At one of these schools, second-year algebra and geometry are taught parallel to each other thus covering two years. This may be the reason for 123 seniors being in the group.

Boys outnumber the girls by approximately two and one-half to one in the junior and senior classes. In the group as a whole the ratio is nearly two to one. Table V shows the class range by sex.

TABLE V - CLASS RANGE BY SEX

	Boys	Girls	Unclassified	Totals
Soph.		1		1
Jun.	187	105	3	295
Sen.	79	43	1	123
P.G.	10			10
N.C.	23	8	2	33
Totals	299	157	6	462

TABLE IV - AGE AND CLASS MARKS FOR GEOMETRIC

	Age 16	14	17	19	Totals
Jun.	1	5			6
Sph.			1		1
H.S.				2	2
Totals	1	4	1	2	8

The third or junior year seems to be the most common time for students to take second-year algebra. At one of these schools, second-year algebra and geometry are taught parallel to each other thus covering two years. This may be the reason for 123 seniors being in the group.

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TABLE V - CLASS RANGE BY SEX

	Boys	Girls	Unknown	Totals
Soph.		1		1
Jun.	187	108	3	298
Sen.	71	45	7	123
Spe.	10			10
H.S.	53	4	8	65
Totals	322	157	8	485

B. VALIDITY

The question of the validity of a test, or of test scores is important. Guilford[1/] has this to say about validity,

> "The question has many facets, and it requires clear thinking not to be confused by them. In crudest terms, we say that a test is valid when it measures what it presumes to measure. This is one shade better than the definition that states that a test is valid if it measures the truth." "... we may say that the question 'Is this test valid?' should be immediately answered by another question, 'It is valid for what?'"

Validity is usually based upon one of two things. They are: (1) a set of criteria which the author has established before he constructs the test, and (2) correlations between the results of a test and other tests.

The author has computed correlations of the results on this test with results on the California Test of Mental Maturity. The California Test is subdivided into Language and Non-Language divisions. The non-language factors are made up of the following aspects: "Immediate Recall, Sensing Right and Left, Manipulation of Areas, Foresight in Spacial Relations, Opposites, Similarities, Analogies, Number Series, and Numerical Quantity."[2/] These may be grouped into three main groups, immediate recall, spacial relationships, and reasoning. The language factors are predominately vocabulary

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1/ Guilford, J. P., Fundamental Statistics in Psychology and Education, McGraw-Hill Book Company, New York, 1942, pp 273.

2/ California Test of Mental Maturity: Advanced Series: Grades 9-Adult, California Test Bureau, Los Angeles, California

B. VALIDITY

The question of the validity of a test, or of test scores is important. Guilford[1] has this to say about validity.

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The author has obtained correlation of the results on this test with results on the California Test of Mental Maturity. The California Test is subdivided into language and non-language divisions. The non-language factors are made up of the following aspects: "Immediate Memory, General Right and Left, Manipulation of Areas, Forward in Spatial Relations, Opposites, Similarities, Analogies, Number Series and Numerical Quantity."[2] These may be grouped into three main groups, immediate recall, spatial relationships, and reasoning. The language factors are broken mainly visualizing

1/ Guilford, J. P., Fundamental Statistics in Psychology and Education. McGraw-Hill Book Company, New York. 1942, pp 279.

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and delayed recall.

With the Non-Language I.Q. a coefficient of correlation of 0.16 was obtained as compared with a 0.55 correlation with the Language I.Q. The former correlation is slight and shows almost negligible relationship.[1] The correlation of 0.55 is a moderate correlation but it shows substantial relationship.[2] From the comparison of these correlations it appears that the algebra test is more dependent upon language ability than upon the non-language abilities.

C. OTHER CORRELATIONS

The scores on the algebra test were also correlated with the General Ability I.Q. of the California Test of Mental Maturity and with school marks.

With the former a correlation of 0.51 was obtained. While this is a moderate correlation, it shows substantial relationship and therefore may be considered important.

School marks correlated 0.66 with the test scores. These marks were given by three teachers in five sections of the class.

Of the 462 results obtained, 50 were used in figuring the correlations which are reported in this chapter. These 50 pupils are members of one of the high schools which gave this test to its mathematics class. This school made its records available with the understanding that the author would not mention the locality. This is the reason for not

1/ Guilford, J. P., Op. Cit., pp 219.
2/ Guilford, J. P., Op. Cit., pp 219.

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With the Non-Language I.Q. a coefficient of correlation of 0.16 was obtained as compared with a 0.55 correlation with the Language I.Q. The former correlation is slight and shows almost negligible relationship.[1] The correlation of 0.55 is a moderate correlation but it shows substantial relationship.[2] From the comparison of these correlations it appears that the algebra test is more correlated with Language ability than upon the non-language aptitude.

C. OTHER CORRELATIONS

The scores on the algebra test were also correlated with the General ability I.Q. of the Kuhlmann Test of Mental Maturity and with school marks.

With the former a correlation of 0.61 was obtained. While this is a moderate correlation, it shows substantial relationship and therefore may be considered important.

School marks correlated 0.66 with the test scores. These marks were given by three teachers in the sections of the class.

Of the 108 regular students, 80 were used in figuring the correlations which are reported in this chapter. These 80 pupils are members of one of the high schools which gave this test to its mathematics class. This school made its records available with the understanding that the author would not mention the locality. While the reasons for not

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[1] Guilford, J. P., op. cit., pp 298.
[2] Guilford, J. P., op. cit., pp 298.

giving written recognition to this school.

All correlations mentioned were figured by use of a scatter diagram and the Pearson r. The formula for the Pearson r is:

$$r_{xy} = \frac{\frac{\Sigma XY}{N} - (C_x C_y)}{(\sigma_x)(\sigma_y)}$$

Where: X and Y are deviations from the guessed mean in terms of the class interval as the unit.

C_x and C_y are corrections in X and Y.

σ_x and σ_y are standard deviations in X and Y in terms of the class interval as the unit.

D. RELIABILITY

The reliability of a test merits considerable consideration since conclusions which are to be drawn should be free from faults.

> "By a reliable test, we mean one that is free from erros of measurment so that successive measurments of the same individual or phenomenon would yield exactly the same values."1/

There are three methods commonly used in correlating a test. They are the (1) test-retest method, (2) alternate forms method, and (3) the split-half method. In order to find a coefficient of reliability for his test, the writer has used the split-half method.

1/Guilford, J. P., Op. Cit., pp 273

giving written representation to this account.

All correlations mentioned were figured by use of a scatter diagram and the Pearson r. The formula for the Pearson r is:

$$r_{xy} = \frac{\frac{\Sigma D_x D_y}{N} - (D_x D_y)}{(\sigma_x)(\sigma_y)}$$

where: X and Y are deviations from the assumed mean in terms of the class interval as the unit.

D_x and D_y are corrections in X and Y.

σ_x and σ_y are standard deviations in X and Y in terms of the class interval as the unit.

D. RELIABILITY

The reliability of a test merits considerable consideration since correlations which are to be drawn should be free from faults.

> "By a reliable test, we mean one that is free from error of measurement so that successive measurements of the same individual or phenomenon would yield exactly the same values."[1]

There are three methods commonly used in correlating a test. They are the (1) test-retest method, (2) alternate form method, and (3) the split-half method. In order to find a coefficient of reliability for this test, the writer has used the split-half method.

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[1] Pritzker, A. E., Op. cit., pp 273

The Spearman-Brown formula for estimating the reliability was used. In using this formula one important requirement is that the two parts should yield equal standard deviations in the same group of testees. The two parts of this test yielded standard deviations of 3.34 and 3.35. The author correlated every other item against the others. This is almost an odd v. even, but, due to the different number of items in the various sections, this is not quite true.

With the Spearman-Brown formula a reliability coefficient of 0.971 was obtained. Table VI shows the scatter diagram and computation used in obtaining this correlation.

The Spearman-Brown formula is:

$$r_t = \frac{2r_{1/2}}{(1 + r_{1/2})}$$

Where: r_t is the self-correlation of a test in its full length.

$r_{1/2}$ is the self-correlation of one-half the test.

X: ONE HALF

Y: OTHER HALF

Scores	0-1	2-3	4-5	6-7	8-9	10-11	12-13	14-15	16-17	18-19	20-21	22-23	24-25	26-27	F_y	Y	FY	FY^2	ΣXY +	ΣXY −
26-27													$^{25}1_{25}$		1	+5	+5	25	25	0
24-25											$^{12}1_{12}$	$^{16}8_{128}$	$^{20}10_{200}$	$^{24}4_{96}$	23	+4	+92	368	436	0
22-23									$^{3}1_{3}$	$^{6}1_{6}$	$^{9}11_{99}$	$^{12}18_{216}$	$^{15}18_{270}$	$^{18}1_{18}$	50	+3	+150	450	612	0
20-21							$^{-2}1_{-2}$	1	$^{2}5_{10}$	$^{4}15_{60}$	$^{6}22_{132}$	$^{8}21_{108}$	$^{10}8_{80}$		73	+2	+146	292	450	2
18-19							$^{-1}3_{-3}$	3	$^{1}14_{14}$	$^{2}27_{54}$	$^{3}17_{51}$	$^{4}4_{16}$	$^{5}1_{5}$		69	+1	+69	69	140	3
16-17						1	6	10	9	12	5				43	0	0	0	0	0
14-15					$^{3}3_{9}$	$^{2}3_{6}$	$^{1}5_{5}$	11	$^{-1}8_{-8}$	$^{-2}2_{-4}$	$^{-3}1_{-3}$	$^{-4}1_{-4}$			34	−1	−34	34	20	19
12-13			$^{10}1_{10}$	$^{8}1_{8}$	$^{6}2_{12}$	$^{4}8_{32}$	$^{2}4_{8}$	6	$^{-2}1_{-2}$						23	−2	−46	92	70	2
10-11				$^{12}3_{36}$	$^{9}8_{72}$	$^{6}8_{48}$	$^{3}4_{12}$	1							24	−3	−72	216	168	0
8-9		$^{24}1_{24}$		$^{16}7_{112}$	$^{12}12_{144}$	$^{8}10_{80}$	$^{4}2_{8}$	1							33	−4	−132	528	368	0
6-7			$^{25}5_{125}$	$^{20}11_{220}$	$^{10}7_{70}$										35	−5	−175	875	595	0
4-5		$^{36}1_{36}$	$^{30}6_{180}$	$^{24}9_{216}$	$^{18}5_{90}$										21	−6	−126	756	522	0
2-3	$^{49}2_{98}$	$^{42}7_{294}$	$^{35}11_{385}$	$^{28}2_{56}$	$^{21}1_{21}$										23	−7	−161	1127	854	0
0-1	$^{56}2_{112}$	$^{48}7_{336}$	$^{40}1_{40}$												10	−8	−80	640	488	0
F_x	4	16	24	33	43	37	25	33	38	57	57	52	38	5	462		−364	5472	4748	26
X	−7	−6	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6						
FX	−28	−96	−120	−132	−129	−74	−25	0	+38	+114	+171	+208	+190	+30	+147					
FX^2	196	576	600	528	387	148	25	0	38	228	513	832	950	180	5201					
ΣXY +	210	690	740	648	528	236	33	0	27	120	294	528	580	114	4748				ΣXY =	+ 4722
ΣXY −	0	0	0	0	0	0	5	0	10	4	3	4	0	0	26					

Spearman–Brown formula for estimating reliability

$$r_t = \frac{2r_{1/2}}{(1+r_{1/2})} = \frac{2(.945)}{(1+.945)} = .971$$

$$r_{xy} = \frac{\frac{\Sigma XY}{N} - (C_x C_y)}{(\sigma_x)(\sigma_y)} = \frac{\frac{4722}{462} - (.318)(-.788)}{(3.34)(3.35)} = .945$$

$$C_x = \frac{\Sigma FX}{N} = .318$$

$$C_y = \frac{\Sigma FY}{N} = -.788$$

$$\sigma_x = \sqrt{\frac{\Sigma FX^2}{N} - (C_x)^2} = 3.34$$

$$\sigma_y = \sqrt{\frac{\Sigma FY^2}{N} - (C_y)^2} = 3.35$$

TABLE VI - SCATTER DIAGRAM FOR RELIABILITY OF TEST

CHAPTER IV

INTERPRETATIONS OF TEST RESULTS

In this chapter, the author will attempt to interpret the results obtained from giving the test in two ways. First, he will discuss the results on the whole test, and secondly, he will discuss the results on the individual sections of the test.

Before doing this, a few words about the method of administering the test are in order. The tests and the answer sheets were placed in the hands of the classroom teacher who gave the test. The General Directions to the test were read by the teacher and then the pupils were allowed to work at their own rates of speed.

As was expected, the groups did not have a standard period of time in which to work on the test. The actual working time ranged from 26 to 45 minutes. Approximately half of the classes had over 40 minutes, while the remainder had from 30 to 35 minutes, with the exception of one 26 minute period.

Because of this great difference in the length of class periods it was feared that many students would not have an opportunity to work on the last few sections of the test. To counteract this, some of the groups began work with Section VII and worked backwards. This did not work out satisfactorily so the tactics were changed and other groups

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As was expected, the groups did not have a standard period of time in which to work on the test. The actual working time ranged from 25 to 45 minutes. Approximately half of the classes had over 40 minutes, while the remainder had from 30 to 35 minutes, with the exception of one 25 minute period.

Because of this great difference in the length of class periods it was feared that many students would not have an opportunity to work on the last few sections of the test. To counteract this, some of the groups began work with section VII and worked backwards. This did not work out satisfactorily as the tactics were changed and other groups

began with Section III and worked to the end of the test, then returning to the first parts. This proved to be the better plan as it did not give the pupil a feeling of "working backwards" as much as starting with Section VII did.

For each discussion concerning the results obtained on this test the author has included at least two tables. These tables are designated as Table X (or some other numeral) A or B. In each case, Table A is the actual number of attempts, correct attempts, and incorrect attempts. Similarly, Table B is the percentages of the same facts. The percentage of incorrect attempts and the percentage of correct attempts are based upon the number of attempts, while the percentage of attempts is based upon 462 (the total number taking the test). The author has based his discussion on Tables B as he believes these tables give a more accurate representation of the situation.

For convenience, the author has indicated the tables pertinent to the section being discussed along with the heading for the section.

A. THE TEST AS A WHOLE

(Table VII, Figure 1, Tables VIII A and VIII B)

Results obtained on the test were quite satisfactory in that they show differences in the results of the various items and sections. It is these differences which show where pupils are encountering trouble in solving word problems.

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A. THE TEST AS A WHOLE

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Table VII gives a brief picture of how the various classes and the entire group did. All scores indicated in the table are arithmetical averages obtained by dividing the total number of correct responses for the particular column by the number taking the test in the group.

The mean score for the entire group of 462 subjects is 29.95. From a glance at Table VII it is apparent that this number does not give a satisfactory value for the whole group. Actually three divisions are indicated. The first of these divisions contains means of 40.43 and above, the second means of 28.43 through 37.25 and the third means of 11.06 through 18.00. The standard deviation for the entire group is 13.08.

There are two reasons for this great variation of mean scores for the groups. They are (1) the time element and (2) some groups started with the sections of the test which provided fewer points toward the total score. In scoring the test, one point was given for each correct answer, a possible total of 54. From the results, it appears that a period of at least 40 minutes is desirable in order to obtain best results.

Individual scores ranged from 1 to 51 with the average score being 29.95. Two girls received a total of 51 points. The most common scores were 37 and 40, there being 46 students evenly divided between these two. Figure 1 is a graphic representation of the individual scores plotted against the

Table VII gives a brief summary of how the various classes and the entire group did. All scores indicated in the table are arithmetical averages obtained by dividing the total number of correct responses for the particular column by the number taking the test in the group.

The mean score for the entire group of 183 subjects is 24.85. From a glance at table VII it is apparent that this number does not give a satisfactory value for the whole group. Normally three divisions are indicated. The first of these divisions contains means of 40.42 and above, the second means of 28.42 through 37.28 and the third means of 11.00 through 18.00. The standard deviation for the entire group is 13.08.

There are two reasons for this great variation of mean scores for the groups. They are (1) the time element and (2) some groups started with the sections of the test which provided fewer points toward the total score. In scoring the test, one point was given for each correct answer, a possible total of 50. From the results, it appears that a period of at least 40 minutes is desirable in order to obtain best results.

Individual scores ranged from 1 to 51 with the average score being 24.85. Two girls received a total of 51 points. The most common scores were 37 and 40, there being 45 students evenly divided between these two. Figure 1 is a graphic representation of the individual scores plotted against the

TABLE VII - RESULTS BY CLASSES

Results of Algebra test on solving word problems.
Students scores are raw scores. Group scores are averages of each group tested.

	~~Student~~/Group	No. St 1/	No. Min.	Section I	II	III	IV	V	VI	VII	Total
	Possible			16	9	10	10	2	2	5	54
1	A	28	45	14.61	7.79	9.21	8.64	0.32	1.29	3.29	43.89
2	B	20	45	14.30	7.75	8.55	8.35	0.35	1.35	2.90	43.40
3	C	26	40	14.58	8.04	8.96	8.50	0.12	0.88	2.23	43.27
4	D	23	42	12.91	7.91	8.48	8.96	0.04	0.87	2.22	41.48
5	E	26	43	14.19	7.77	8.23	8.41	0	0.58	1.77	41.12
6	F	23	44	13.65	8.13	7.70	7.96	0	0.78	1.96	40.03
7	G	4	43	12.00	7.00	7.75	8.00	0	0.25	2.25	37.25
8	H	21	41	13.52	7.24	7.28	8.00	0.05	0.14	0.80	36.76
9	I	14	42	13.21	7.07	6.93	8.14	0.07	0.21	0.93	36.57
10	J	26	35	12.77	7.38	7.93	6.58	0	0.31	0.23	35.31
11	K	23	26	11.65	7.22	7.65	5.43	0	0.09	2/	32.22
12	L	32	33	10.16	7.22	7.28	6.47	0	0.06	0.13	31.38
13	M	17	33	9.94	6.59	6.94	5.47	0	0.12	0.41	29.47
14	N	30	35	8.80	6.83	6.77	5.47	0	0.07	0	28.43
15	O 3/	26	32	1.15	1.12	7.15	7.27	0	0.24	1.00	18.00
16	P 4/	27	32	5.81	4.33	3.59	2.19	0	0.07	0.37	16.22
17	Q 3/	19	33	1.32	0.42	6.68	7.37	0	0	0.11	15.74
18	R 4/	21	33	5.10	3.19	2.29	1.81	0	0.24	2.57	15.19
19	S 3/	24	32	1.42	1.58	4.33	4.00	0.04	0.37	1.79	13.54
20	T 4/	32	33	0.19	1.63	2.75	3.34	0	0.44	2.78	11.06
21	Totals	462		9.31	5.73	6.74	6.37	0.05	0.43	1.36	29.95

22 1/ Number of students

23 2/ No attempts

24 3/ Started with Section III and worked to end of test then returned to first sections

25 4/ Started with Section VII and worked backwards

26 There is a key available for the lettering of the Groups if desired

27

28 This is the same information that was returned to the teachers. They also received individual results of the pupils in their classes

1	A	38	46	14.41	7.76	8.21	8.64	0.38	7.47	5.29	42.09
2	B	20	46	14.30	7.76	8.88	0.89	0.50	0.35	2.50	15.40
3	C	23	40	14.58	8.04	8.76	0.60	0.12	0.66	3.28	43.27
4	D	23	42	12.01	7.61	6.48	6.56	0.04	0.67	2.58	41.44
5	E	26	43	14.19	7.77	8.23	8.41	0	0.66	1.77	47.12
6	F	28	44	12.88	8.13	7.70	7.96	0	0.76	1.90	49.03
7	G	4	43	12.00	7.00	7.76	8.00	0	0.48	2.98	37.45
8	H	21	41	13.88	7.24	7.28	8.00	0.08	0.14	3.07	38.76
9	I	14	42	12.21	7.07	6.92	8.14	0.07	0.21	0.35	38.27
10	J	28	38	12.77	7.32	7.98	6.88	0.[illegible]	0.31	0.23	38.21
11	K	23	35	11.68	7.22	7.68	6.43	0	0.09	[illegible]	32.32
12	L	32	33	10.16	7.32	7.83	6.47	0	0.06	0.15	27.38
13	M	17	33	9.[illegible]	6.49	4.94	6.47	0	0.12	0.41	32.47
14	N	30	35	8.00	6.82	6.77	6.47	0	0.07	0	28.42
15	O	26	32	1.15	1.12	7.15	7.27	0	0.32	1.00	10.00
16	P	27	32	5.91	4.32	4.39	2.15	0	0.07	0.27	25.32
17	Q	19	33	1.12	0.42	6.68	7.37	0	0	0.11	15.74
18	R	21	35	3.10	3.15	5.29	1.27	0	0.24	3.57	15.15
19	S	24	32	1.42	1.59	4.63	4.00	0.04	0.27	1.75	13.44
20	T	29	33	0.19	1.63	5.75	3.34	0	0.45	3.75	12.06
21	Total	456		9.31	5.73	6.74	6.37	0.05	0.43	1.55	29.38

1/ Number of students

2/ No attempts

3/ Started with Sections III and worked to One of Least Cows
Started on first exercise

4/ Started with section VII and worked backwards

Points in a key available for the identing of the wrong
in Samples

This is the same information that was included in the
Samples. By a method (initial) results of the

TABLE VIII A - FIGURES FOR ENTIRE TEST

Section	Attempts	Correct	Incorrect
I	4996	4294	702
II	3218	2652	566
III	3815	3115	700
IV	3561	2968	593
V	480	23	457
VI	406	199	207
VII	888	633	255
Totals	17364	13884	3480

TABLE VIII B - PERCENTAGES FOR ENTIRE TEST

Section	Percentage of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
I	67.7	82.4	17.6
II	77.4	82.4	17.6
III	82.6	81.9	18.1
IV	77.1	83.4	16.6
V	52.0	4.8	95.2
VI	44.0	49.0	51.0
VII	38.4	71.3	29.7
Totals	69.6	79.9	20.1

frequency of these scores.

The fact that the curve is bi-modal stands out in the graph. It can be seen that a gradual increase occurs from

1 to 16 then the curve drops slowly to 26 and rises again more sharply than before. The left peak of the curve represents the scores of the pupils who started with Section III or VII and thus did not have an opportunity to make as large a score as the others. The right half of the curve has a slight protrusion at the beginning of the ascent which represents the scores of the students starting with Section I but having less than 40 minutes testing time. The remainder of the curve represents the scores of the pupils who had 40 or more minutes to work on the test.

From these results it appears that the students are well prepared in most of the steps of solving word problems. Only in the last three sections are serious discrepancies encountered. These steps are perhaps the most important ones of all. It may be an indication that the pupils should receive better instructions in these phases of problem solving. More will be said about the various sections of the test in the next part of this chapter.

B. THE INDIVIDUAL SECTIONS

I. Vocabulary (Tables IX A and IX B)

Upon checking the results of this section, it was discovered that approximately 82 per cent of the attempts were correct. This leads to the conclusion that pupils are well prepared in the vocabulary needed in the solution of woed problems.

1 to 16 then the curve drops slowly to 25 and rises again more sharply than before. The left peak of the curve represents the scores of the pupils who started with section III or VII and thus did not have an opportunity to make as large a score as the others. The right half of the curve has a slight protrusion at the beginning of the second which represents the scores of the students starting with section 1 but having less than 40 minutes testing time. The remainder of the curve represents the scores of the pupils who had 40 or more minutes to work on the test.

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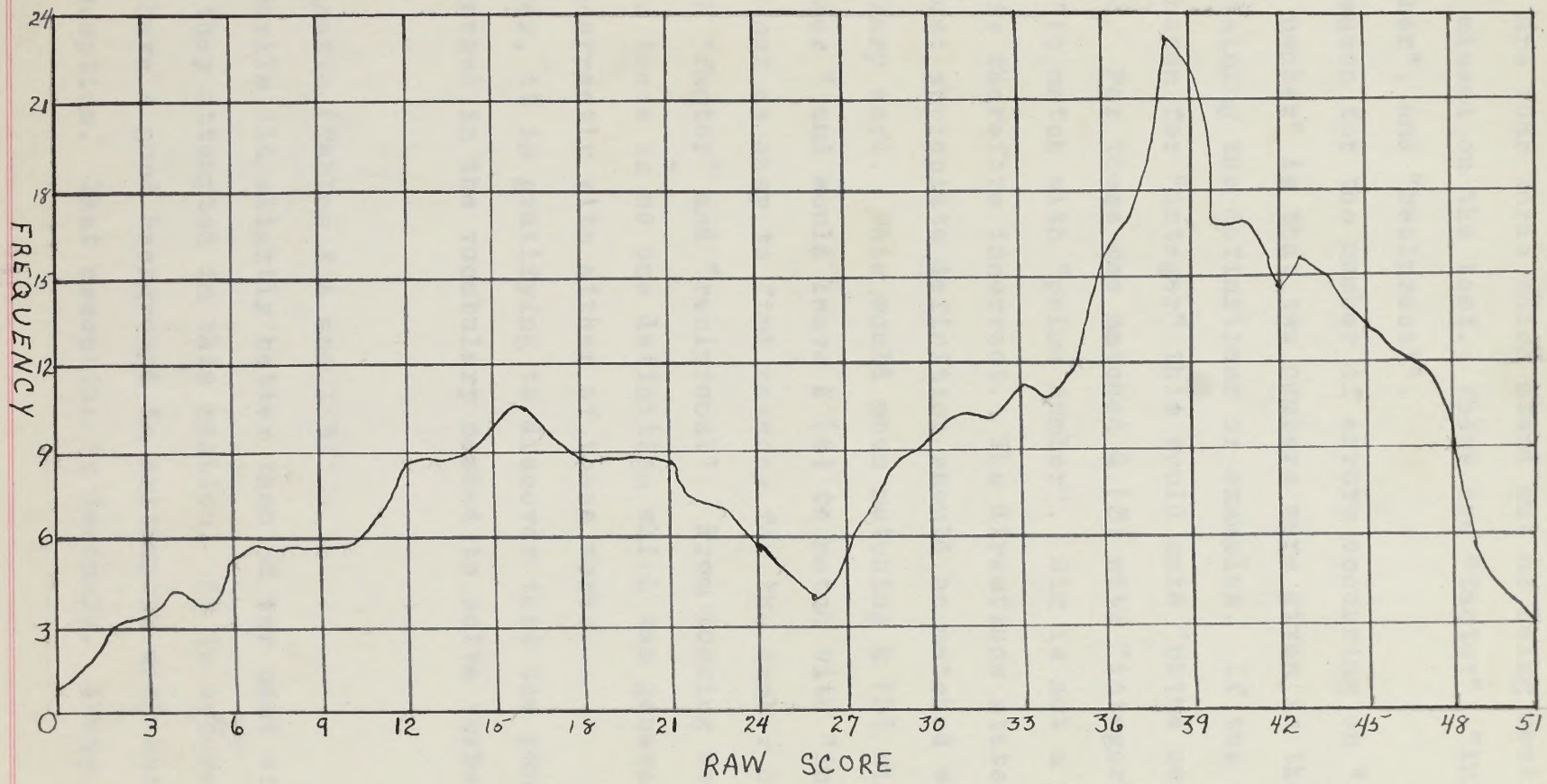

This graph is a smoothed average adapted from graph of actual results

FIGURE 1 - FREQUENCY OF SCORES

There are four words which stand out as being most frequently missed on the test. These are "factor", "integer", "prime number", and "reciprocal".

The reason for the number of errors occurring on "integer" and "prime number" is that two numbers were given in the column containing the definitions or examples. If the wrong one were chosen for "integer" this would make "prime number" wrong, also. For those who matched Q (3) with "integer" had N (6) left to match with "prime number". Six is not a prime number so is therefore incorrect. The directions stated that the most appropiate definition should be matched with the vocabulary word. This would mean matching Q (3) with "prime number " and would leave N (6) to match with "integer".

It is not as easy to find reasons for the number of errors with "factor" and "reciprocal". From looking over the test papers there is no one definition which was generally matched incorrectly with either of these words.

However, it is gratifying to discover that the pupils are well versed in the vocabulary needed to solve verbal problems.

II. Arithmetic (Tables X A and X B)

The pupils did slightly better than 82 per cent of the exercises they attempted in this section. It is apparent that they have a good background in arithmetic with but one notable exception. That exception is decimals. Among the

There are four words which stand out as being most frequently missed on the test. These are "factor", "integer", "prime number", and "reciprocal".

The reason for the number of errors occurring on "integer" and "prime number" is that two numbers were given in the column containing the definitions or examples. If the wrong one were chosen for "integer" this would make "prime number" wrong, also. For those who matched 4 (3) with "integer" had 8 (6) left to match with "prime number". 8 is not a prime number so is therefore incorrect. The directions state that the most appropriate definition should be matched with the vocabulary word. This would mean matching 4 (3) with "prime number " and would leave 8 (6) to match with "integer".

It is not so easy to find reasons for the number of errors with "factor" and "reciprocal". From looking over the test papers there is no one definition which was generally matched incorrectly with either of these words.

However, it is gratifying to discover that the pupils are well versed in the vocabulary needed to solve verbal problems.

ii. Arithmetic (Tables X A and I B)

The pupils did slightly better than 50 per cent of the exercises they attempted in this section. It is apparent that they have a good background in arithmetic with but one notable exception. That exception is decimals. Among the

TABLE IX A - FIGURES FOR SECTION I

Number	Attempts	Correct	Incorrect
1	356	338	18
2	354	338	16
3	315	270	45
4	334	316	18
5	352	350	2
6	335	325	10
7	343	333	10
8	303	266	37
9	305	283	22
10	294	254	40
11	303	291	12
12	282	198	84
13	310	291	19
14	261	139	122
15	289	142	147
16	260	160	100
Totals	4996	4294	702

TABLE IX B - PERCENTAGES FOR SECTION I

Number	Percentage of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
1	77.1	94.9	5.1
2	76.6	95.5	4.5
3	68.2	85.7	14.3
4	72.3	94.6	5.4
5	76.2	99.4	0.6
6	72.5	97.0	3.0
7	74.2	97.1	2.9
8	65.6	87.8	12.2
9	66.0	92.8	7.2
10	63.6	86.4	13.6
11	65.6	96.0	4.0
12	61.0	70.2	29.8
13	67.1	93.9	6.1
14	56.5	53.3	46.7
15	62.6	49.1	50.9
16	56.3	61.5	39.5
Totals	67.7	82.4	17.6

most often missed examples of the group were numbers 3 and 4 which are multiplying and dividing decimals. The one large error here occurred in placing the decimal point in the correct place. Many of those who attempted the problems had the correct figures but did not place the decimal point in the correct place.

TABLE X A - FIGURES FOR SECTION II

Number	Attempts	Correct	Incorrect
1	370	327	43
2	372	340	32
3	365	277	88
4	360	285	75
5	357	283	74
6	353	296	57
7	350	290	60
8	347	308	39
9	344	246	98
Totals	3218	2652	566

TABLE X B - PERCENTAGES FOR SECTION II

Number	Percentage of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
1	80.1	88.4	11.6
2	80.5	91.5	8.5
3	79.0	75.9	24.1
4	77.9	79.2	20.8
5	77.3	79.3	20.7
6	76.4	83.9	16.1
7	75.8	82.9	17.1
8	75.1	88.8	11.2
9	74.5	71.5	28.5
Totals	77.4	82.4	17.6

Another common mistake occurred in number 5 which is the addition of two fractions. Some missed this by not

most often missed examples of the group were numbers 6 and 8 which are multiplying and dividing decimals. The one large error here occurred in placing the decimal point in the correct place. Many of those who attempted the problems had the correct figures but did not place the decimal point in the correct place.

TABLE I A - PROBLEM TYPE SECTION II

Number	Attempts	Correct	Incorrect
1	371	327	45
2	374	320	52
3	366	277	89
4	365	290	75
5	357	283	74
6	352	325	27
7	350	293	60
8	347	300	47
9	344	254	90
Totals	3216	2552	664

TABLE I B - PERCENTAGES FOR SECTION II

Number	Percentage of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
1	80.1	88.2	11.8
2	80.8	91.8	8.8
3	79.0	75.8	24.1
4	77.9	79.5	20.5
5	77.8	79.3	20.7
6	76.4	85.3	14.7
7	75.8	83.5	16.1
8	75.1	86.5	13.5
9	74.5	73.8	26.2
Totals	77.4	79.1	20.8

Another common mistake observed in number 3 which is the addition of two fractions. Some missed this by not

obtaining the correct denominator while others tried to find the solution by multiplying the two numbers.

The largest number of errors on this section occurred with the last example. This was a straight division problem. Many tried to do it with decimals while others tried to reduce the fraction obtained. In the directions they were asked to leave the remainders as fractions.

III. Algebra Fundamentals (Tables XI A and XI B)

With this section, the author discovered a good background in the fundamentals of algebra. Among the ten examples chosen for this section, two caused some difficulty, while two others caused a great deal more difficulty. The two causing some difficulty are numbers 4 and 9, while numbers 7 and 10 caused a great deal of difficulty.

Example number four is the subtraction of a negative number from a positive number. Not changing the sign was the main reason that many missed this. Number nine is an example in factoring. The expression to be factored is $2X^2 + X - 6$. The solution is $(2X - 3)(X + 2)$. Many interchanged the signs or some of the numbers in their solutions. One person wrote "not factorable" on his answer sheet for this problem.

Example number seven is the addition of two algebraic fractions which equal 26. Many missed this because of not obtaining the correct denominator or because they failed to

obtaining the correct denominator while others tried to find the solution by multiplying the two numbers.

The largest number of errors on this section occurred with the last example. This was a difficult division problem. Many tried to do it with logarithms while others tried to reduce the fraction obtained. In the directions they were asked to leave the remainders as fractions.

III. Algebra readiness (Problem 11 a and 11 b)

With this section, the author discovered a good background in the fundamentals of algebra. Among the two examples chosen for this section, two caused some difficulty, while two others caused a great deal more difficulty. The two causing some difficulty are numbers 6 and 9, while numbers 7 and 10 caused a great deal of difficulty.

Example number four is the subtraction of a negative number from a positive number. Not changing the sign was the main reason that many missed this. Number nine is an example in factoring. The expression to be factored is $2x^2 + x - 6$. The solution is $(2x - 3)(x + 2)$. Many interchanged the signs or some of the numbers in their solutions. One person wrote "not factorable" on his answer sheet for this problem.

Example number seven is the addition of two algebraic fractions which equal 2x. Many missed this because of not obtaining the correct denominator or because they failed to

multiply both sides of the equation by 12. It is interesting to note that the addition of fractions in both the arithmetic section and the section on algebra fundamentals caused trouble.

TABLE XI A - FIGURES FOR SECTION III

Number	Attempts	Correct	Incorrect
1	407	386	21
2	407	384	23
3	407	392	15
4	407	297	110
5	405	372	33
6	405	375	30
7	346	196	150
8	393	335	58
9	368	266	102
10	270	112	158
Totals	3815	3115	700

TABLE XI B - PERCENTAGES FOR SECTION III

Number	Percentage of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
1	88.1	94.8	5.2
2	88.1	94.4	5.6
3	88.1	96.3	3.7
4	88.1	73.0	17.0
5	87.7	91.9	8.1
6	87.7	92.6	7.4
7	74.9	56.7	43.3
8	85.1	85.2	14.8
9	79.7	72.3	27.7
10	58.4	41.5	58.5
Totals	82.6	81.9	18.1

The author included one formula to solve in this group. It was this problem which the greatest number of pupils missed. Only 41.5 per cent of those attempting this exercise obtained the correct answer. There were many different

multiply both sides of the equation by 18. It is interesting to note that the addition of fractions in both the arithmetic section and the section on algebra fundamentals seemed trouble.

TABLE XI A - FIGURES FOR SECTION III

Number	Attempts	Correct	Incorrect
1	407	388	41
2	407	384	23
3	407	386	18
4	407	287	110
5	405	372	33
6	405	275	20
7	346	196	150
8	393	338	55
9	258	155	103
10	270	112	156
Totals	3815	3115	700

TABLE XI B - PERCENTAGES FOR SECTION III

Number of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts	Number
1	95.1	94.5	5.2
2	88.1	94.4	5.8
3	88.1	94.3	4.7
4	88.1	73.0	17.0
5	87.7	91.9	8.2
6	87.7	92.5	7.4
7	74.9	56.7	43.3
8	85.1	85.8	14.2
9	79.7	72.3	27.7
10	58.4	41.5	58.5
Totals	82.5	81.9	18.1

The author included one formula to solve in this group. It was this problem which the greatest number of pupils missed. Only 41.5 per cent of those attempting this exercise obtained the correct answer. There were many different

incorrect answers given, typical of these were: $N = \frac{1}{CR}$, $N = \frac{N}{C + R}$, and others similar to these. It appears that classes in algebra have not had adequate instruction in the solution of formulas. This is an ability that will be needed when the student takes courses in chemistry and physics and should be well taught in the algebra classes.

IV. Reading (Tables XII A and XII B)

It was on this section that the students compiled their best record on the test. They answered 83.4 per cent of their attempts correctly. There are two possible reasons for this high ratio of correct attempts. Perhaps the pupils can read problems and obtain the desired information. The other alternative may be that this section of the test is poorly constructed and that the problems and questions did not present enough difficulty to the student.

There is only one case in which the majority of the pupils failed in its attempt to answer the question. That was the last statement which required the reciprocal to the number stated in the problem. It is notable that a majority failed this item and that the word "reciprocal" was one of the most frequently missed words in the vocabulary section.

From the results obtained with this test, however, it seems to indicate that pupils are able to read verbal problems and locate information about them without too much difficulty.

Incorrect answers given, typical of those were: $S_n = \frac{1}{d^n}$, $S = \frac{M}{d + n}$, and others similar to these. It appears that students in algebra have not the adequate instruction in the solution of formulas. This is an ability that will be needed when the student takes courses in chemistry and physics and should be well taught in the algebra classes.

IV. Reading (Tables XII A and XII B)

It was on this section that the students surprised their best record on the test. They answered 82.4 per cent of their attempts correctly. There are two possible reasons for this high ratio of correct attempts. Perhaps the pupils can read problems and obtain the desired information. The other alternative may be that this section of the test is merely correctional and that the problems and questions did not present enough difficulty to the student.

There is only one case in which the majority of the pupils failed in its attempt to answer the question. That was the last statement which required the reciprocal to the number stated in the problem. It is notable that a majority failed this item and that the word "reciprocal" was one of the most frequently misused words in the vocabulary section.

From the results obtained with this test, however, it seems to indicate that pupils are able to read verbal problems and locate information about them without too much difficulty.

TABLE XII A - FIGURES FOR SECTION IV

Number	Attempts	Correct	Incorrect
1.-A	374	332	42
-B	383	350	33
2.-A	381	316	65
-B	376	326	50
3.-A	370	323	47
-B	365	345	20
4.-A	350	319	31
-B	341	297	44
5.-A	329	309	20
-B	292	51	241
Totals	3561	2968	593

TABLE XII B - PERCENTAGES FOR SECTION IV

Number	Percentage of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
1.-A	81.0	88.2	11.8
-B	82.9	91.4	8.6
2.-A	82.5	82.9	17.1
-B	81.4	86.7	13.3
3.-A	80.1	87.3	12.7
-B	79.0	94.5	5.5
4.-A	75.8	91.1	8.9
-B	73.8	87.1	12.9
5.-A	71.2	93.9	6.1
-B	63.2	17.5	82.5
Totals	77.1	83.4	16.6

V. Setting up Algebraic Expressions (Tables XIII A and XIII B)

This was the section of the test from which the poorest results were obtained. Therefore, the conclusion drawn is that this step in the solution of word problems is causing the greatest difficulty for algebra students.

One reason for the low percentage of correct attempts on the first problem is because it contains two unknowns.

TABLE XII A - RESULTS FOR SECTION IV

Number	Attempts	Correct	Incorrect
1.-A	271	232	42
-B	285	260	25
2.-A	241	213	28
-B	276	246	30
3.-A	270	223	47
-B	268	243	25
4.-A	280	219	61
-B	241	207	44
5.-A	229	209	20
-B	225	191	34
Totals	3521	2564	957

TABLE XII B - RESULTS FOR SECTION IV

Number of attempts	Percentage of Correct Attempts	Percentage of Incorrect attempts
1.-A	81.0	88.2	11.8
-B	88.9	91.4	8.6
2.-A	88.3	88.3	11.7
-B	81.4	86.7	13.3
3.-A	80.1	82.6	17.4
-B	72.0	90.3	9.7
4.-A	78.2	81.1	18.9
-B	72.8	87.1	12.9
5.-A	74.8	91.3	8.7
-B	85.3	17.8	82.2
Totals	77.1	86.4	13.6

V. Testing by algebraic expressions (Tables XIII A and XIII B)

This was the section of the test from which the poorest results were obtained. Therefore, the conclusion drawn is that this step in the solution of word problems is causing the greatest difficulty for algebra students.

One reason for the low percentage of correct attempts on the first problem is because it contains two unknowns.

The majority of pupils attempting this problem tried to set up expressions in one unknown--usually that of the boat's rate of speed. Their reason for doing this may be that the example given was in one unknown.

TABLE XIII A - FIGURES FOR SECTION V

Number	Attempts	Correct	Incorrect
1	259	3	256
2	221	20	201
Totals	480	23	457

TABLE XIII B - PERCENTAGES FOR SECTION V

Number	Percentage of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
1	56.1	1.2	98.8
2	47.8	9.1	90.9
Totals	52.0	4.8	95.2

Another reason for the great number of errors in this section may be that the problems were not as familar to the students as some of the other types of word problems are. One of the teachers reported that she did not place much emphasis upon up-and-down-stream problems.

Because this section was placed after the four sections which had the greatest amount of reading and figuring, it is possible that the student became rushed for time when he reached this part of the test so began to hurry in order to finish. If this assumption is true then this may be a reason for the poor performance.

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TABLE XIII B - PERCENTAGES FOR SECTION V

Number of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
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Another reason for the great number of errors in this section may be that the problems were not as familiar to the students as some of the other types of word problems are. One of the teachers reported that she did not place much emphasis upon up-and-down-stream problems.

Because this section was placed after the four sections which had the greatest amount of reading and figuring, it is possible that the student became tired for this when he reached this part of the test so began to hurry in order to finish. If this assumption is true then this may be a reason for the poor performance.

There are two interesting facts to note about this section. The first is that only one student set up correct expressions for each of the problems. The other is that one pupil wrote, "Can't do worded problems" on his answer sheet in the space provided for him to write the expressions.

VI. Setting up Equations (Tables XIV A and XIV B)

From this section, the author found that approximately one-half of the attempts to set up equations were correct. However, within the section a large discrepancy occurs. The first equation was properly set up by over 64 per cent of those trying while in the second problem approximately three-quarters of the attempts resulted in failure.

TABLE XIV A - FIGURES FOR SECTION VI

Number	Attempts	Correct	Incorrect
1	252	162	90
2	154	37	117
Totals	406	199	207

TABLE XIV B - PERCENTAGES FOR SECTION VI

Number	Percentage of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
1	54.6	64.3	35.7
2	33.3	24.0	76.0
Totals	44.0	49.0	51.0

The most common errors in the first equations were: $\frac{X}{12} + 9 = \frac{X}{6}$ or $\frac{X}{6} + 9 = \frac{X}{12}$. The correct equation for this

There are two interesting facts to note about this section. The first is that only one student got no correct experiences for each of the problems. The other is that one small area, "don't do, correct procedure" on his inventory seemed to be areas provided for him to write his experiences.

VI. Setting up Equations (Items XIV A and XIV B)

From this section, the author found that approximately one-half of the attempts to set up equations were correct. However, within the section a large discrepancy occurs. The first question was properly set up by over 64 per cent of those trying while in the second problem approximately three-quarters of the attempts resulted in mistakes.

TABLE XIV A - FIGURES FOR SECTION VI

Number	Attempts	Correct	Incorrect
1	238	162	76
2	164	27	137
Totals	402	189	213

TABLE XIV B - FIGURES FOR SECTION VI

Number	Percentage of attempts	Percentage of correct attempts	Percentage of incorrect attempts
1	59.2	68.0	35.7
2	40.8	32.0	76.0
Totals	74.0	42.0	81.0

The most common errors in the first equations were: $\frac{4}{12} + 2 = \frac{4}{8}$ or $\frac{4}{8} + 2 = \frac{4}{12}$. The correct solution for this

problem is $\frac{X}{12} + \frac{X}{6} = 9$ (or this information in some other arrangment).

In the second problem there were numerous attempts to set up an equation. The correct equation should be: $1/4(60 + X) = 6 + X$ (or this information in some other arrangment). The two most common mistakes were: $1/4(60 + X) = X$ and $1/4(60 + X) = 6$. In general, it may be stated that they did not realize that the two equal expressions should be set equal to each other in the equation.

While it may not be safe to draw too general conclusions from such a limited sample, it appears that more instruction should be given on setting up equations.

VII. Solving Equations (Tables XV A and XV B)

Results on this section of the test are not as good as those for Sections I through IV, but they are much better than those for Sections V and VI. It appears that students do fairly well in solving equations of the types included in the test, but that more practice is needed in the solution of these kinds of equations.

Among the reasons for missing the first problem were the interchanging of the prices of tomatoes and peaches. This is the only one which difinitely showed up on the answer sheets. If the paper upon which the pupils did their work were available other reasons could possibly be discovered.

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TABLE XV A - FIGURES FOR SECTION VII

Number	Attempts	Correct	Incorrect
1.-A	224	148	76
-B	217	157	60
2	227	166	61
3.-A	111	82	29
-B	109	80	29
Totals	888	633	255

TABLE XV B - PERCENTAGES FOR SECTION VII

Number	Percentage of Attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
1-A	48.5	66.1	33.9
-B	47.0	70.4	29.6
2	49.1	73.1	26.9
3-A	24.0	73.9	26.1
-B	23.6	73.4	26.6
Totals	38.4	71.3	28.7

The reason for missing the third problem is probably because the students did not know how to apply the quadratic formula in this case. Another possible reason is that they did not know how to extract square roots.

For the second problem no reason for the errors showed up on the answer sheets. There were a variety of incorrect answers, but the author is unable to state any reason for these.

TABLE IV A - FIGURES FOR SECTION VII

Number	Attempts	Correct	Incorrect
1.-A	224	148	76
-2	217	137	80
2	227	166	61
3.-A	111	83	28
-B	109	80	29
Totals	838	583	255

TABLE IV B - PERCENTAGES FOR SECTION VII

Number	Percentage of attempts	Percentage of Correct Attempts	Percentage of Incorrect Attempts
1-A	48.6	66.1	33.9
-B	47.0	70.4	29.6
2	49.1	73.1	26.9
3-A	24.0	73.9	26.1
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CHAPTER V

SUMMARY AND CONCLUSIONS

In this thesis the author has considered the problem of discovering where pupils make mistakes in word problems in algebra. To accomplish this it became necessary to construct a test to be used in compiling data for this report.

This test was given to 462 students representing six schools in Massachusetts and Vermont. The composition of this group has been analyzed in Chapter III of this thesis.

The writer has discussed the validity and reliability of the test and in addition has given other correlations which were obtained. Chapter II contains a discription of the test as a whole as well as a discussion of the individual sections which go to make up the testing instrument.

In Chapter IV is presented the results obtained from the administering of the test with interpretations. Tables have been used extensively in this chapter in order to give a true representation of the results item by item as well as by sections and of the test in its entirety. To discuss each item would lead to confusion and unnecessary work since the topic under consideration is locating difficulties. The tables can tell the story about those items which seem to offer little difficulty.

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From the results of the test, the author drawes several

important conclusions. These conclusions are:

(1) Setting up algebraic expressions appears to be the most difficult step for pupils to perform.

(2) Setting up algebraic equations appears to be the next most difficult step for students.

(3) The solution of simultaneous and quadratic equations is a common difficulty. Approximately 25 per cent of the group failed to solve these equations correctly.

(4) The use of decimals is troublesome to nearly one-quarter of the pupils. Not placing the decimal point in the correct place is the most common error.

(5) The addition of fractions in both arithmetic and algebra proved troublesome. Approximately 20 per cent of the students missed in the arithmetic and 45 per cent missed in the algebra.

(6) There is a tendency to confuse the words "integer" and "prime number". Teachers should make the meanings of these terms clear.

(7) The meaning of "reciprocal" is not known by 40 per cent of the pupils. Almost 83 per cent did not know how to use the reciprocal of a number when asked to do so.

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(6) There is a tendency to confuse the words "integer" and "prime number". Teachers should make the meanings of these terms clear.

(7) The meaning of "reciprocal" is not known by 40 per cent of the pupils. Almost 85 per cent did not know how to use the reciprocal of a number when asked to do so.

(8) The pupils in general have good backgrounds in vocabulary, arithmetic, algebra fundamentals, and reading.

CHAPTER VI

RECOMMENDATIONS

At this time the author will make some recommendations to various persons concerned with the teaching of verbal problems. Among these are the teacher, the supervisor, the curriculum planner, and those in schools of education.

The teacher is the person who has the most influence upon the pupil and is therefore the one with whom these recommendations will be most concerned. The teacher in the elementary grades has not succeeded in giving the pupil lasting instruction in the use of decimals and the addition of fractions. These teachers should give more thorough instructions covering these topics.

In the high school, the algebra teacher should review the backgrounds of his pupils and reteach those areas which appear to be below normal. In the algebra work the pupils should be given more practice in the solution of formulas and the addition of algebraic fractions.

The setting up of algrbraic expressions should be given more emphasis in order that the student will see why certain expressions should receive values which are different from other expressions. He should also be able to decide which expression is to be used as a basis for comparison with the other expressions. Since this step appears to cause the greatest difficulty in problem solving, adequare instruction

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The setting up of algebraic expressions should be given more emphasis in order that the student will see why certain expressions should receive values which are different from other expressions. It should also be able to decide which expression is to be used as a basis for comparison with the other expressions. Since this step appears to cause the greatest difficulty in problem solving, elementary instruction

on this point may help a great deal in the successful solution of word problems. More practice should also be allotted to the setting up of equations and the solution of equations.

The supervisors should be held responsible for seeing that the classroom teachers receive whatever help is needed to put better teaching methods into operation. Curriculum planners should allow more time for the teaching of problem solution. Much of this time should be spent on setting up algebraic expressions and equations.

Our schools of education in their methods of teaching courses should place more emphasis on the solution of verbal problems. This would give potential teachers of mathematics an insight into some of the difficulties they will encounter. Since many teachers do not receive adequate experience as student teachers before they enter the classroom such a procedure seems to be highly desirable.

The setting up of algebraic expressions appears to be the point at which most emphasis should be placed by all persons concerned. It is this step which causes the most difficulty and should receive more and better instruction.

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APPENDICIES

A - The Test

B - The Answer Sheet

C - Key to the Test

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General Directions

Remove the answer sheet from this booklet and fill in the blanks with the required information. Put all answers on the answer sheet. Do NOT write in this booklet. Use the blank sheet of paper for figuring. If more paper is needed use the back of the Answer sheet.

When the command to start is given you are to commence working and to complete as many of the exercises as possible in the time you have. If one section seems to give you toruble leave it and go on to another. Come back to the harder ones after you have completed those you can do easily.

SECTION I--VOCABULARY

Directions

In column A is a list of terms found in Algebra. Column B contains a list of definitions. Match the definition in Column B with the term in column A. There may be more than one definition for some of the terms. If so, choose the most appropriate definition. Do not use any letter more than once. Match letters A to L with Numbers 1 to 8 and letters M to U with numbers 9 to 16. Number O has been done as an example.

Column A.	Column B
0. (C) Trinomial	A. Result obtained by division.
1. Product	B. One variable varies inversly with the other.
2. Quotient	C. 8y + 2f - 11i.
3. Direct variation	D. Every 3 years.
4. Bi-annually	E. Result obtained by multiplication.
5. Sum	F. One variable varies directly with the other.
6. Per cent	G. Proportion of parts of a hundred.
7. Equal	H. Has a fixed value.
8. Constant	I. Having the same value as another quantity.
9. Formula	J. Result obtained by addition.
10. Proportion	K. Result obtained by subtraction.
11. Consecutive	L. Every 2 years.
12. Factor	M. Terms which follow directly after one another.
13. Fraction	N. 6.
14. Integer	P. $A = \frac{1}{2}Bh$.
15. Prime number	Q. 3.
16. Reciprocal	R. One of 2 or more terms used in forming a product.
	S. 8/3.
	T. $\frac{S_1}{A_1} = \frac{S_2}{A_2}$.
	U. The answer when the number 1 is divided by a number.

SECTION II--ARITHMETIC

Directions
Solve the following examples by performing the indicated operations. Show all remainders as fractions.

1. $\begin{array}{r} 3682 \\ +8753 \\ \underline{1940} \end{array}$ 2. $\begin{array}{r} 7351 \\ \underline{-3678} \end{array}$ 3. $\begin{array}{r} 81.7 \\ \underline{\times 36.1} \end{array}$ 4. $.45\overline{)9.04590}$ 5. $\frac{7}{8} + \frac{2}{3}$

6. $\frac{5}{6} - \frac{3}{5}$ 7. $\frac{9}{24} + \frac{8}{3}$ 8. $\frac{7}{8} \times \frac{5}{6}$ 9. $24\overline{)5379}$

SECTION III--ALGEBRA (Fundamentals)

Directions
Find answers to the following exercises by performing the indicated operations.

1. $15Y = 225$
   $Y =$
2. $X + 3 = 27$
   $X =$
3. $b - 14 = 2$
   $b =$
4. $(+6)-(-8) =$
5. $(-2)(-4) =$
6. $(-3)(+6) =$
7. $\frac{3X}{4} + \frac{X}{3} = 26$
   $X =$
8. $(X + 4)(X - 2) =$
9. Factor: $2X^2 + X - 6$
10. Solve for N: $\frac{1}{N} = \frac{1}{C} + \frac{1}{R}$

SECTION IV--READING

Directions
Read each problem throughly and then complete the statements following the problem. Put your answers on the answer sheet. Do NOT solve the problem in order to complete the statements.

0. (Example): The first angle of a triangle is twice as large as the second angle and the third angle is three times as large as the second angle.
Statements: A. The third angle is <u>1 1/2</u> times as large as the first angle.
B. The <u>2nd</u> angle is the smallest angle of the triangle.

1. A stenographer requires 30 hours to type a certain manuscript. A second stenographer, who takes 2 minutes more to each page, requires 40 hours to finish the same manuscript.
Statements: A. The first stenographer types a page in 2 minutes _____ time than the second.
B. It takes the second stenographer ____ hours longer to type the manuscript.

2. A field has four sides of uneq ual lengths and the distance around the field is 112 rods. The first side is twice the length of the second side, the third side is 10 rods longer than the second side, and the fourth side is 12 rods shorter than the

Section IV-Con't

second side.

Statements: A. The ____ side is the shortest side.
B. The second side is ____ the size of the first side.

3. A commuter returning by train from his office ordinarily reaches his suburban station at 5 o'clock. His chauffeur leaves his home with a car just in time to meet the man when the train arrives, and drives him back home. On a certain day the man takes a train which arrives at the station 1 hour earlier. He walks toward home until he meets his chauffeur and car. The chauffeur turns and drives the man the rest of the way home, arriving home 20 minutes earlier than usual.

Statements:. A. By taking the earlier train the man was able to arrive home _____ sooner than he ordinarily would have.
B. The man arrived at the station at ____ o'clock.

4. A 10-gallon cask contains pure alcohol. Five gallons are drawn off and replaced by water; then 5 gallons of the mixture are drawn off and replaced by water, and so on until 8 drawings and 8 replacements have been made.

Statements: A. In all ____ dra wings are made.
B. After the last replacement the cask contains ____ gallons of the mixture.

5. A man owns a horse, a carriage worth $100 more than the horse, and a harness. The horse and harness are together worth three-fourths the value of the carriage, and the carriage and harness are together worth $50 less than twice the value of the horse.

Sta tements: A. The horse is owth $100 ____ than the carriage.
B. The carriage is worth _____ times as much as the horse and harness.

SECTION V--SETTING UP ALGEBRAIC EXPRESSIONS

Directions

Set up the values you would give the unknown quatities needed to find the answer in the following exercises. Do NOT set up the equation.

0. (Example): You will be given this informa tion:
A father is three times as old as his son. Five years ago he was 5 times as old as his son was then. What are their present ages?

You are to do this:
X = Son's age
$3X$ = Father's age
$X - 5$ = Son's age 5 years ago
$3X - 5$ = Father's age 5 years ago

1. A motor boat goes 36 miles down a river in 2 hours and comes back the same distance in 3 hours. Find the boat's own rate.

2. How many ounces of water must be added to a mixture of 1 ounce of ammonia and 10 ounces of water to make a solution that is 1% ammonia?

SECTION VI--SETTING UP EQUATIONS

Directions

From the information given you are to set up the equation needed to solve the problem. Do NOT solve the equation.

0. (Example): You will be given this information:

A grocer wishes to produce a 42-cent coffee by mixing a 40-cent coffee with a 60-cent coffee. How much of each grade should he use to make 45 pounds of the mixture?

N = Number of pounds of 40-cent coffee
$45 - N$ = Number of pounds of 60-cent coffee
$40N$ = Cost of 40-cent coffee in cents
$(60)(45 - N)$ = Cost of 60-cent coffee in cents
$(42)(45)$ = Cost of mixture in cents

You are to do this: Equation: $40N + (60)(45 - N) = (42)(45)$

1. A boat can go 12 miles an hour downstream and 6 miles an hour upstream. Some boys hire the boat to go down the river as far as they can go and still get back in 9 hours. How many miles down the river can they go?

X = Number of miles
$\frac{X}{12}$ = Number of hours to go downstream
$\frac{X}{6}$ = Number of hours to go upstream
9 = Number of hours

Equation: _____

2. A man wishes to add enough alcohol to 60 quarts of a 10% solution to form a 25% solutions. How many quarts of alcohol must he add?

X = Number of quarts of alcohol added
$60 + X$ = Number of quarts in new solution
$(1/4)(60 + X)$ = Number of quarts of alcohol in new solution
$(1/10)(60)$ = Number of quarts of alcohol in old solution
6 = Number of quarts of alcohol in old solution
$6 + X$ = Number of quarts of alcohol in new solution

Equation: _____

SECTION VII--SOLVING EQUATIONS

Directions

From the information given and the equation set up find the answer to the question asked in the problem.

0. (Example): You will be given information similar to this:

The sum of two consecutive numbers is 55. Find the numbers.

X = First number
$X + 1$ = Second number
55 = Sum of the numbers

Equation: $X + X + 1 = 55$

You are to do this: $2X = 54$
$X = 27$
$X + 1 = 28$

27 1st No.

28 2nd No.

1. I can buy 6 cans of peaches and 12 cans of tomatoes for \$1.68, or 12 cans of peaches and 6 cans of tomatoes for \$1.92. What is the price per can of each?

X = Price of 1 can of peaches
Y = Price of 1 can of tomatoes
$6X$ = Price of 6 cans of peaches
$12Y$ = Price of 12 cans of tomatoes
$12X$ = Price of 12 cans of peaches
$6Y$ = Price of 6 cans of tomatoes

Equations: $6X + 12Y = 168$
$12X + 6Y = 192$

2. One-half hour after an aviator, flying at the rate of 110 miles an hour, left a certain place, another airplane started from the same place to overtake him. How long will it take the second aviator, at the rate of 130 miles an hour to overtake the first?

h = Number of hours needed
$h + 1/2$ = Number of hours first plane travels
$110(h + 1/2)$ = Number of miles first plane travels
$130h$ = Number of miles second plane travels

Equation: $130h = 110(h + 1/2)$

3. The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

X = First odd number
$X + 2$ = Second odd number
394 = Sum of the squares of the numbers

Equation: $X^2 + (X + 2)^2 = 394$

ANSWER SHEET

Name_____ Age_____ Date_____ 194_____
Last First Middle Initial

School_____ City_____ State_____

Class_____ Section_____ Have you taken Algebra before this Year__?

Section I	Section II	Section III	Section IV
1....( )	1._____	1._____	1.A._____
2....( )			
3....( )	2._____	2._____	B._____
4....( )			
5....( )	3._____	3._____	2.A._____
6....( )			
7....( )	4._____	4._____	B._____
8....( )			
9....( )	5._____	5._____	3.A._____
10....( )			
11....( )	6._____	6._____	B._____
12....( )			
13....( )	7._____	7._____	4.A._____
14....( )			
15....( )	8._____	8._____	B._____
16....( )			
	9._____	9._____	5.A._____
		10._____	B._____

Section V

1.

2.

Section VI

1.

2.

Section VII

1._____Peaches

_____Tomatoes

2._____Hours

3._____1st no.

_____2nd no.

KEY TO TEST

Section I (Vocabulary)	Section II (Arithmetic)	Section III (Algebra)	Section IV (Reading)
1..(E)	1...14375	1...15	1.A..Less
2..(A)			B..10
3..(F)	2...3673	2...24	
4..(L)			2.A..4th
5..(J)	3...2949.37	3...16	B..1/2
6..(G)			
7..(I)	4...20.102	4...14	3.A..20 Min.
8..(H)			B.. 4
9..(P)	5...$\frac{37}{24}$	5... 8	
10..(T)			4.A.. 8
11..(M)		6...-18	B..10
12..(R)	6...$\frac{7}{30}$		
13..(S)		7...24	5.A..Less
14..(N)			B..4/3
15..(Q)	7...$\frac{9}{64}$	8...$X^2 + 2X - 8$	
16..(U)			
		9...$(2X - 3)(X + 2)$	
	8...$\frac{35}{48}$	10..$N = \frac{CR}{C + R}$	
	9...224 1/8		

Section V
(Setting up algebraic expressions)

1. X = Boat's rate
   Y = Rate of Current
   $X + Y$ = Rate downstream
   $X - Y$ = Rate upstream
   $2(X + Y)$ = Number of miles downstream
   $3(X - Y)$ = Number of miles upstream

2. X = Number of ounces of water added
   11 = Number of ounces in first solution
   $X + 11$ = Number of ounces in new solution
   $(1/100)(11 + X)$ = Number of ounces of ammonia in new solution
   1 = number of ounces of ammonia in first solution

Section VI
(Setting up equations)

1. $\frac{X}{12} + \frac{X}{6} = 9$

2. $(1/4)(60 + X) = 6 + X$

Section VII
(Solving equations)

1...12¢ Peaches
    8¢ Tomatos

2...2 3/4 Hours

3...13 or -15 First number
    15 or -13 Second number

KEY TO TEST

Section I (Vocabulary)	Section II (Arithmetic)	Section III (Algebra)	Section IV (Reading)
1...(E)	1...14575	1...15	1.A...Less
2...(A)			2...10
3...(F)	2...3573	2...24	
4...(D)			3.A...4th
5...(C)	3...2945.37	3...16	3...1/2
6...(G)			
7...(I)	4...20.102	4...14	3.A...20 Min.
8...(B)			3... 4
9...(J)	5...$\frac{27}{64}$	5... 5	
10...(T)			4.A... 8
11...(M)		6...-18	4...10
12...(R)	6...$\frac{7}{20}$		
13...(S)		7...54	5.A...Less
14...(N)			5...4/3
15...(Q)	7...$\frac{9}{64}$	8...$X^2 + 6X - 8$	
16...(U)			
		9...(2X -3)(X+2)	
	8...$\frac{25}{64}$		
		10...N = $\frac{6X}{X+1}$	
	9...221 1/9		

Section V
(Setting up algebraic expressions)

1. X = Boat's rate
   Y = Rate of current
   X+Y = Rate downstream
   X - Y = Rate upstream
   3(X + Y) = Number of miles downstream
   3(X - Y) = Number of miles upstream

2. X = Number of ounces of water added
   11 = number of ounces in first solution
   X+11 = Number of ounces in new solution
   (1/100)(11 + X) = Number of ounces of ammonia in new solution
   1 = number of ounces of ammonia in first solution

Section VI
(Setting up equations)

1. $\frac{X}{12} \cdot \frac{X}{5} = 9$

2. (1/4)(50 + X) = 5 + X

Section VII
(Solving equations)

1...12½ Peaches
   8½ Tomatoes

2...2 3/4 Lemons

3...15 or -1 [illegible]
   [illegible]

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